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A NOTE ON CONSTRUCTING BALANCED INCOMPLETE BLOCK DESIGNS

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SUMMARY

A method has been described for constructing a class of balanced incomplete block (BIB) designs by using mutually orthogonal latin squares (MOLS) in which the diagonal elements are in natural order.

Keywords : Orthogonal latin squares, Balanced incomplete block desings.

Introduction

Muller [1] introduced a method of constructing a class of balanced incomplete block (BIB) designs with parameters :

$$v = s \quad b = s(s-1) \quad r = t(s-1) \quad k = t \text{ and } \lambda = t(t-1) \qquad (1)$$

(where $t < s$) whenever a complete set of mutually orthogonal latin squares (MOLS) of order s exists. Consequently his method is applicable only when s is a prime or prime power. Recently Subramani [2] has developed an alternative method of constructing BIB designs with parameters (1) by using only a set of t MOLS. This can be true for any value of s. For example, if there exists a set of 4 MOLS of order 10, then we can construct a BIB design with parameters

$$v = 10 \quad b = 90 \quad r = 36 \quad k = 4 \text{ and } \lambda = 12 \qquad (2)$$

whereas Muller's method requires a set of 9 MOLS of order 10. In this paper, further reduction on the requirement of the MOLS has been made to construct the BIB designs (1) under certain conditions. This procedure requires minimum number of MOLS comparing to the methods of Subramani [2] and Muller [1]. For example, the procedure to be discussed in Section 2, requires only 2 MOLS to construct the BIB design (2), as demonstrated by means of an example using two MOLS of order 10 with diagonal elements in natural order in each of them, whereas the methods of Subramani [2] and Muller [1] essentially need 4 and 9 MOLS respectively.

2. Main Results

Throughout this note we use the notations $0\ (n, t)$ and $H\ (n, m)$ to represent respectively the existence of t MOLS and mth order hyper-graeco latin squares (HGLS) of order n. To refer readily the method of Subramani [2] is given below :

Let $L\ 1, L\ 2, \ldots, Lt$ be a set of t mutually orthogonal latin squares (MOLS) of order n with the elements in the first row in natural order. That is, MOLS are all in semistandard form. By superimposing any m MOLS we get mth order HGLS $H\ (n,m)$ of order n. Then omitting the first row and forming the remaining $n\ (n - 1)$ cells as blocks we get a class of BIB designs with parameters

$$v = n \quad b = n\ (n - 1) \quad r = m\ (n - 1) \quad k = m \text{ and } \lambda = m\ (m - 1) \quad (3)$$

where $2 \leqslant m \leqslant t$.

An alternative method which needs only $(m - 2)$ MOLS is as follows :

Let $L\ 1, L\ 2, \ldots, Lt$ be a set of t MOLS with main diagonal elements in natural order. If we superimpose any m MOLS we get HGLS $H\ (n, m)$. The element in (r, s)th cell of $H\ (n, m)$ is denoted by $hrs = (i\ 1, i\ 2, \ldots, im)$ where ij is the element corresponding to the jth square. Obviously we can have

$$i\,1 = i\,2 = \ldots = im \quad \text{if } r = s$$
$$i\,1 \neq i\,2 \neq \ldots \neq im \quad \text{if } r \neq s$$

That is the diagonal elements of $H\ (n, m)$ are in natural order.

By omitting the diagonal cells and forming the remaining cells together with their cell representation as a' block we have $n\ (n - 1)$ blocks each of size $(m + 2)$. For example, if we have the elements $(i1\ i2 \ldots im)$ in (r, s)th cell then the block obtained from this is $(r\ s\ i1\ i2 \ldots im)$. Similarly we can obtain the other blocks. It can be easily shown that these $n\ (n - 1)$ blocks constitute a BIB design with parameters

$$v = n \quad b = n\,(n-1) \quad k = m+2 \quad r = (m+2)\,(n-1) \text{ and}$$
$$\lambda = (m+2)\,(m+1) \tag{4}$$

where $2 \leqslant m \leqslant i$.

Since each element is associated with all other elements equally in $(m+2)\,(m+1)$ cells and occurs in $(m+2)\,(n-1)$ cells the proof follows. To illustrate explicitly, consider the pair $(r\ s)$. By this procedure, rth row is associated with sth element in m cells and sth row is associated with rth element in m cells. Similarly, rth and sth columns are associated with sth and rth elements respectively in m cells. Further in the $H\,(n, m)$, the pair $(r\ s)$ occurs in $m\,(m-1)$ cells the rth row is associated with sth column and sth row is associated with rth column. Totally pair $(r\ s)$ comes $4m + m\,(m-1) + 2 = (m+2)\,(m+1)$ times. And the other conditions are obvious by the definition of $O\,(n, t)$ and $H\,(n, m)$.

The following theorem summarizes the above result.

Theorem 1 : *If $H\,(n, t)$ exists with diagonal elements in natural order then we can always construct a BIBD with parameters*

$$v = n \quad b = n\,(n-1) \quad k = m+2 \quad r = (m+2)\,(n-1) \text{ and}$$
$$\lambda = (m+2)\,(m+1) \tag{5}$$

where $2 \leqslant m \leqslant t$.

Corollary 1 : Let $t = 1$ in $H\,(n, t)$. That is, if there exists only one latin square with diagonal elements in natural order then the resulting BIB design is

$$v = n \quad b = n\,(n-1) \quad k = 3\,(n-1) \quad \text{and } \lambda = 6 \tag{6}$$

It is to be noted that the above design always exists for any integral value of $n > 3$.

3. Example

Let us consider two orthogonal latin squares $L\,1$ and $L\,2$ of order 10 as given below.

$L\,1$

1	3	6	9	7	4	2	0	8	5
9	2	4	7	1	8	5	3	0	6
0	1	3	5	8	2	9	6	4	7
5	0	2	4	6	9	3	1	7	8
8	6	0	3	5	7	1	4	2	9
3	9	7	0	4	6	8	2	5	1
6	4	1	8	0	5	7	9	3	2
4	7	5	2	9	0	6	8	1	3
2	5	8	6	3	1	0	7	9	4
7	8	9	1	2	3	4	5	6	0

$L\,2$

1	9	0	5	8	3	6	4	2	7
3	2	1	0	6	9	4	7	5	8
6	4	3	2	0	7	1	5	8	9
9	7	5	4	3	0	8	2	6	1
7	1	8	6	5	4	0	9	3	2
4	8	2	9	7	6	5	0	1	3
2	5	9	3	1	8	7	6	0	4
0	3	6	1	4	2	9	8	7	5
8	0	4	7	2	5	3	1	9	6
5	6	7	8	9	1	2	3	4	0

By superimposing $L1$ and $L2$ and omitting the diagonal cells, the remaining cells together with their cell representation constitute the BIBD with parameters

$$v = 10 \quad b = 90 \quad r = 36 \quad k = 4 \quad \text{and} \quad \lambda = 12$$

1239	2193	3106	4159	5187	6134	7162	8140	9128	0175
1360	2341	3214	4207	5261	6298	7245	8273	9250	0286
1495	2470	3452	4325	5308	6372	7319	8356	9384	0397
1578	2516	3580	4563	5436	6409	7483	8421	9467	0418
1643	2689	3627	4690	5674	6547	7501	8594	9532	0529
1726	2754	3791	4738	5710	6785	7658	8602	9615	0631
1804	2837	3865	4812	5849	6820	7896	8769	9703	0742
1982	2905	3948	4976	5923	6951	7930	8917	9871	0853
1057	2068	3079	4081	5092	6013	7024	8085	9046	0964

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[1] Muller, E. R. (1965) : A method of constructing balanced incomplete block designs, *Biometrika*, 52 : 285-288.

[2] Subramani, J. (1988) : A method of construction of balanced incomplete block designs (in press).